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CREA Business Cycle Indexes: the Methodology

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CREA Business Cycle Indexes: the Methodology*

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Abstract

We modify the framework developed by Aruoba, Diebold, and Scotti (2009) for measuring economic activity at high frequency by using changes of mixed-frequency variables instead of their levels. This modification enables to avoid using polynomial detrending and to better handle data revision.

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1 The Dynamic Factor Model (DFM)

Let $Y_{i,t}$ denote the monthly (daily) value of i-th economic variable at month (day) t. We remove its trend by considering the k-month (k-day) change $\Delta_k Y_{i,t} = Y_{i,t} - Y_{i,t-k}$. We assume that the dynamics of $\Delta_k Y_{i,t}$ is governed by its p lags and a monthly (daily) unobserved common factor b_t:

$$\Delta_k Y_{i,t} = c_i + \lambda_i b_t + \sum_{j=1}^{p} \rho_{ij} \Delta_k Y_{i,t-n_i j} + u_{i,t} \tag{1}$$

where $u_{i,t}$ are contemporaneously and serially uncorrelated white noises with variance σ_i^2. n_i is the number of months (days) per observational period. For instance, if Y_i is collected every quarter, then n_i will be equal to 3 in the case of a monthly Business Cycle Index (BCI) b_t and to 90, 91 or 92 for an index at the daily frequency. Hence, notice that n_i is time-varying in the case of a daily index for monthly and quarterly collected data. To simplify notations, it is assumed that n_i is fixed (as for an index at the monthly frequency), but in the state-space representation, it will be treated as time-varying. As in Aruoba, Diebold, and Scotti (2009), lags of $\Delta_k Y_{i,t}$ in multiple of n_i are also is also introduced in equation (1) since assuming persistence only at the monthly (daily) frequency would not be appropriate as it would disappear prematurely for variables at a lower frequency.
Our Business Cycle Index (BCI) b_t captures the cyclical dynamics of a Swiss canton or of the national economy at the highest frequency available in the dataset (i.e., daily or monthly). This variable follows an auto-regressive process of order r:

$$b_t = \phi_1 b_{t-1} + ... + \phi_r b_{t-r} + \epsilon_t \tag{2}$$

where ϵ_t is assumed to be a white noise innovation with mean 0 and variance $\sigma_\epsilon^2 = 1 - \sum_{j=1}^{r} \phi_j^2$. The model is thus composed of measurement equations (1) for each observed variable $i = 1, ..., N$ expressed in k-month (k-day) change, and a transition equation (2). The selection of order k depends on the base frequency of observation and on the highest frequency available of the dataset. We will discuss it in detail in next section. We will also derive the function form of measurement that depends on the type of variable (stock versus flow) as well as its frequency. We will then show how to cast the model in state-space form and how to implement it to extract the BCI b_t.

1.1 Measurements for Flow and Stock Variables in the Monthly DFM

Let $t = 1, ..., T$ index months, where T is the total number of months in the sample. Let Y_{i,q_t} denote the quarterly value of i-th variable at monthy t. In case of a flow variable such at GDP, it is equal to the sum of their unobserved monthly values $Y_{i,t}$:

$$Y_{i,q_t} = Y_{i,t} + Y_{i,t-1} + Y_{i,t-2}$$

As the dependent variable of measurement equation (1) is expressed in k-months change, we now have to choose the order of change. A natural candidate would be $k = 12$ so that we work with year-on-year change. As key advantages of such a choice, we obtain a smoother timeseries and we do not need to filter out the seasonal pattern of the original data. The main drawback is that it does not reflect the most recent information since year-on-year change is simply the sum of the quarter-on-quarter changes over the past 12 quarters. The lowest order that we can select is $k = 3$ because the base frequency of observation is monthly. We thus face a tradeoff between smoothness and responsiveness to new information. Since each quarter has always 3 months, we have:

$$\Delta_{k(q)} Y_{i,q_t} = Y_{i,q_t} - Y_{i,q_t - k(q)} = \Delta_k Y_{i,t} + \Delta_k Y_{i,t-1} + \Delta_k Y_{i,t-2}$$

where $k(q) \in \{1, 4\}$ is consistent with $k \in \{3, 12\}$. In case of a stock variable such as employment, we have $Y_{i,q_t} = Y_{i,t}$ such that $\Delta_{k(q)} Y_{i,q_t} = \Delta_k Y_{i,t}$.
Given that quarterly variables are only observed the last month of the quarter (i.e., in March, June, September and December), the $k(q)$-quarter change of Y_{i,q_t} is given by

$$\Delta_{k(q)} Y_{i,q_t} = \begin{cases} \sum_{j=0}^{2} \Delta_k Y_{i,t-j} & \text{if } i \text{ is a flow variable and } t \text{ is the last month of the quarter} \\ \Delta_k Y_{i,t}, & \text{if } i \text{ is a stock variable and } t \text{ is the last month of the quarter} \\ NA & \text{otherwise.} \end{cases} \tag{3}$$

Thus, plugging (1) into (3) yields measurement for quarterly variables:

$$\Delta_{k(q)} Y_{i,q_t} = \begin{cases} c_i^* + \lambda_i \sum_{j=0}^{2} b_{t-j} + \sum_{j=1}^{p} \rho_{ij} \Delta_{k(q)} Y_{i,q_t - j} + u_{i,t}^* & \text{if } i \text{ is a quarterly flow variable} \\ & \text{that is observed} \\ c_i + \lambda_i b_t + \sum_{j=1}^{p} \rho_{ij} \Delta_{k(q)} Y_{i,q_t - j} + u_{i,t} & \text{if } i \text{ is a quarterly stock variable} \\ & \text{that is observed} \\ NA & \text{if } i \text{ is a quarterly variable} \\ & \text{that is not observed,} \end{cases} \tag{4}$$

since $\Delta_k Y_{i,q_t - j} = \sum_{j=0}^{2} \Delta_k Y_{i,t-3j}$ is observed the last month of each quarter. Note that $c_i^* = 3c_i$ and $u_{i,t}^* = \sum_{j=0}^{2} u_{i,t-j}$. Even though $u_{i,t}^*$ follows a MA(2) process with variance $\sigma_i^{*2} = 3\sigma_i^2$, it can still be considered as a white noise since it is serially uncorrelated at the observational frequency. Whether i is a stock or a flow variable is a stock or a flow, a monthy variable is always observed and equal to $Y_{i,t}$. Thus, given $\Delta_k Y_{i,m_t} = \Delta_k Y_{i,t}$ for all t, measurement (1) for monthly variables becomes:

$$\Delta_k Y_{i,m_t} = c_i + \lambda_i b_t + \sum_{j=1}^{p} \rho_{ij} \Delta_k Y_{i,m_t - j} + u_{i,t} \tag{5}$$

where $\Delta_k Y_{i,q_t - jn_i} = \Delta_k Y_{i,t-j}$ since it is observed every month for all $j = 1, ..., p$.
The monthly model is characterized by transition equation (2) and measurement equations (4) and (5).

1.2 Measurements for Flow and Stock Variables in the Daily DFM

Let $t = 1, ..., T$ index days, where T is the total number of days in the sample. To simplify the analysis, we distribute the value of the 29th of Frebruary across days of every leap year such that every year has the same number of days (i.e., 365). This surplus day adjustement implies that every month and quarter have the same number of days over the years. Since the number of days per month (quarter) change from one month (quarter) to the other, we choose the order of change $k = 365$ so that we consider year-on-year changes.

Let Y_{i,q_t} denote the quarterly value of i-th variable observed the last day t of the quarter. In case of a flow variable such at GDP, it is equal to the sum of their unobserved daily values $Y_{i,t}$:

$$Y_{i,q_t} = Y_{i,t} + Y_{i,t-1} + \ldots + Y_{i,t-d(q_t)+1}$$

where $d(q_t) = \{90, 91, 92\}$ is the the number of days in quarter q_t. Since each quarter has the same number of days as the same quarter of the preceding year (that is, $d(q_t) = d(q_t - 4)$), the year-on-year change of quarterly flow variables is:

$$\Delta_4 Y_{i,q_t} = Y_{i,q_t} - Y_{i,q_t-4} = \Delta_{365} Y_{i,t} + \Delta_{365} Y_{i,t-1} + \ldots + \Delta_{365} Y_{i,t-d(q_t)+1}$$

In case of a stock variable such as employment, we have $Y_{i,q_t} = Y_{i,t}$ such that $\Delta_4 Y_{i,q_t} = \Delta_{365} Y_{i,t}$. Given that quarterly variables are only observed the last day of the quarter, the year-on-year change of Y_{i,q_t} is given by

$$\Delta_4 Y_{i,q_t} = \begin{cases} \sum_{j=0}^{d(q_t)-1} \Delta_{365} Y_{i,t} & \text{if } i \text{ is a flow variable and } t \text{ is the last day of the quarter} \\ \Delta_{365} Y_{i,t} & \text{if } i \text{ is a stock variable and } t \text{ is the last day of the quarter} \\ NA & \text{otherwise.} \end{cases} \tag{6}$$

Thus, plugging (1) into (6) yields measurement for quarterly variables:

$$\Delta_4 Y_{i,q_t} = \begin{cases} c_i^* + \lambda_i \sum_{j=0}^{d(q_t)-1} b_{t-j} + \sum_{j=1}^{p} \rho_{ij} \Delta_4 Y_{i,q_t-j} + u_{i,t}^* & \text{if } i \text{ is a quarterly flow variable} \\ & \text{that is observed} \\ c_i + \lambda_i b_t + \sum_{j=1}^{p} \rho_{ij} \Delta_4 Y_{i,q_t-j} + u_{i,t} & \text{if } i \text{ is a quarterly stock variable} \\ & \text{that is observed} \\ NA & \text{if } i \text{ is a quarterly variable} \\ & \text{that is not observed,} \end{cases} \tag{7}$$

since $\Delta_4 Y_{i,q_t-j} = \sum_{j=0}^{d(q_t)-1} \Delta_{365} Y_{i,t-365j}$ is observed the last day of each quarter. Note that $c_i^* = d(q_t) c_i$ and $u_{i,t}^* = \sum_{j=0}^{d(q_t)-1} u_{i,t-j}$. Even though $u_{i,t}^*$ follows a $MA(d(q_t) - 1)$ process with variance $\sigma_i^{*2} = d(q_t)\sigma_i^2$, it can still be considered as a white noise since it is serially uncorrelated at the observational frequency.

Let Y_{i,m_t} (Y_{i,w_t}) denote the monthly (weekly) value of i-th variable observed the last day t of the month (week). Similarly, measurement for monthly variables is given by:

$$\Delta_{12}Y_{i,m_t} = \begin{cases} c_i^* + \lambda_i \sum_{j=0}^{d(m_t)-1} b_{t-j} + \sum_{j=1}^{p} \rho_{ij}\Delta_{12}Y_{i,m_t-j} + u_{i,t}^* & \text{if } i \text{ is a monthly flow variable} \\ & \text{that is observed} \\ c_i + \lambda_i b_t + \sum_{j=1}^{p} \rho_{ij}\Delta_{12}Y_{i,m_t-j} + u_{i,t} & \text{if } i \text{ is a monthly stock variable} \\ & \text{that is observed} \\ NA & \text{if } i \text{ is a monthly variable} \\ & \text{that is not observed,} \end{cases} \tag{8}$$

where $d(m_t) = \{28, 30, 31\}$ is the the number of days in month m_t, while measurement for weekly variables becomes:[1]

$$\Delta_{52}Y_{i,w_t} = \begin{cases} c_i^* + \lambda_i \sum_{j=0}^{d(w_t)-1} b_{t-j} + \sum_{j=1}^{p} \rho_{ij}\Delta_{52}Y_{i,w_t-j} + u_{i,t}^* & \text{if } i \text{ is a weekly flow variable} \\ & \text{that is observed} \\ c_i + \lambda_i b_t + \sum_{j=1}^{p} \rho_{ij}\Delta_{52}Y_{i,t-d(w_t)j} + u_{i,t} & \text{if } i \text{ is a weekly stock variable} \\ & \text{that is observed} \\ NA & \text{if } i \text{ a weekly variable} \\ & \text{that is not observed,} \end{cases} \tag{9}$$

where $d(w_t) = 7$ is the the number of days in week w_t. Whether i is a stock or a flow variable is a stock or a flow, a daily variable is always observed and equal to $Y_{i,t}$. Thus, measurement (1) for daily variables becomes:

$$\Delta_{365}Y_{i,d_t} = c_i + \lambda_i b_t + \sum_{j=1}^{p} \rho_{ij}\Delta_{365}Y_{i,d_t-j} + u_{i,t} \tag{10}$$

where $\Delta_{365}Y_{i,d_t-j} = \Delta_{365}Y_{i,t-j}$ since it is observed every day for all $j = 1, ..., p$.
The monthly model is characterized by transition equation (2) and measurement equations (7), (8), (9) and (10).

2 State Space Representation

The dynamic factor model represented by equations (1), (3), and (5) can be formulated in a state-space form as follows. Let y_t denotes a $(n \times 1)$ vector of variables observed at date t. The dynamic that governs y_t can be described in terms of latent variables in a vector ξ $(r \times 1)$ called the state vector. Hence, the state-space representation of the dynamic of y is given by the following system

[1]Note that the year-on-year change considered for weekly variables is $\Delta_{364}Y_{i,t}$ since there is gap of $364 = 7 \times 52$ days between the sunday of a given week and that of the same week of the preceding year.

of equations :

$$\xi_{t+1} = F\xi_t + v_{t+1} \tag{11}$$

$$y_t = A'x_t + H'\xi_t + w_t \tag{12}$$

Where F, A' and H' are matrices of parameters of dimension $(r \times r)$, $(n \times k)$ and $(n \times r)$, respectively, and x_t is a $(k \times 1)$ vector of exogenous variables. Equation 6 is known as the State Equation, and equation 7 is known as the Observation Equation.
The vector v_t $(r \times 1)$ and the vector w_t $(n \times 1)$ are considered white noises :

$$E(v_t v'_\tau) = \begin{cases} Q & \text{, For } t = \tau \\ 0 & \text{, Otherwise} \end{cases} \tag{13}$$

$$E(w_t w'_\tau) = \begin{cases} R & \text{, For } t = \tau \\ 0 & \text{, Otherwise} \end{cases} \tag{14}$$

Where Q and R are $(r \times r)$ and $(n \times n)$ matrices, respectively. The noises v_t and w_t are assumed to be uncorrelated across all periods :

$$E(v_t w'_\tau) = 0 \text{ , for all } t \text{ and } \tau \tag{15}$$

The fact that x_t is exogenous implies that x_t provides no information about ξ_{t+s} or w_{t+s} for any positive s beyond the information already contained in $y_{t-1}, y_{t-2}, ..., y_1$.

3 Kalman Filter and Signal Extraction

The Kalman filter is an algorithm providing estimates of latent variables using observed measurements over time by sequentially updating a linear projection for a dynamic system. It allows the computation of accurate finite-sample forecasts, the exact likelihood function and VAR estimation with parameters that change over time. Based on the observations contained in the vectors y_t and x_t, the components' values of the state vector are estimated through date t.

$$\xi_{t+1|t} \equiv E[\xi_{t+1}|Y_t] \tag{16}$$

where

$$Y_t = (y_t, y_{t-1}, \cdots, y_1, x_t, x_{t-1}, \cdots, x_1)' \tag{17}$$

and $E[\xi_{t+1}|Y_t]$ denotes the linear projections of ξ_{t+1} on Y_t and a constant. The Kalman filter computes these forecasts recursively , generating a serie of state vectors $\xi_{1|0}$, $\xi_{2|1}$, $\cdots$, $\xi_{T|T-1}$. Associated with each of these forecasts is a mean squared error (MSE) matrix, represented by the following $(r \times r)$ matrix :

$$P_{t+1|t} \equiv E[(\xi_{t+1} - \xi_{t+1|t})(\xi_{t+1} - \xi_{t+1|t})'] \tag{18}$$

The recursion starts with $\xi_{1|0}$, which denotes a forecast of ξ_1 based on no observations of y or x. Thus, this is simply the unconditional mean of ξ_1,

$$\xi_{1|0} = E(\xi_1) \tag{19}$$

with the associated MSE :

$$P_{1|0} = E\{[\xi_1 - E(\xi_1)][\xi_1 - E(\xi_1)]'\} \tag{20}$$

Following Hamilton (1994), the subsequent starting values are used to initiate the recursion :

$$\xi_{1|0} = 0$$

$P_{1|0}$ the $(r \times r)$ matrix whose elements are expressed as a column vector is given by :

$$vec(P_{1|0}) = [I_{r^2} - (F \otimes F)]^{-1} vec(Q)$$

Given the starting values $\xi_{1|0}$ and $P_{1|0}$, the next step is to compute $\hat{y}_{1|0}$ and $Var(\hat{y}_{1|0})$.

$$\hat{y}_{1|0} = A'x_1 + H'\xi_{1|0} \tag{21}$$

$$Var(\hat{y}_{1|0}) = V_{y_{1|0}} = H'P_{1|0}H + R \tag{22}$$

For the remainder of the paper, ψ_t represents the surprise between the real value of the process (y_t) and the Kalman filter estimation $(\hat{y}_{t|t-1})$. This surprise could either come from measurement noises w_t or from the movement of $\xi_{t|t-1}$ to $\xi_{t|t}$ during the updating step of the recursion.
K_t represents the gain matrix which will convert the surprise esperance of the movement of ξ_t. For instance, using an extreme case where there is no measurement noises $(w_t = 0)$ then the gain matrix K_t will completely allocate the surprise to the movement of ξ_t and thus the filter will be able to retrieve perfectly the path of ξ_t as a result of perfect observations. The next stage is, therefore, the computation of the surprise ψ_1 and the gain matrix K_1.

$$\psi_1 = (y_1 - A'x_1 - H'\xi_{1|0}) \tag{23}$$

$$K_1 = P_{1|0}H(H'P_{1|0}H + R)^{-1} \tag{24}$$

Finally, the estimate of the state vector $\xi_{1|1}$ can be computed with the associated MSE $P_{1|1}$.

$$\xi_{1|1} = \xi_{1|0} + K_1\psi_1 \tag{25}$$

$$P_{1|1} = P_{1|0} - K_1 H' P_{1|0} \tag{26}$$

The calculations for t = 2,3,..., T all have the same steps so that it will be described in general terms for step t.

$$\begin{aligned}
&\text{Prediction stage}\\
\xi_{t|t-1} &= F\xi_{t-1|t-1}\\
P_{t|t-1} &= FP_{t-1|t-1}F' + Q\\
\hat{y}_{t|t-1} &= A'x_t + H'\xi_{t|t-1}\\
Var(\hat{y}_{t|t-1}) = V_{y_{t|t-1}} &= H'P_{t|t-1}H + R\\
&\text{Updating stage}\\
\psi_t &= (y_t - A'x_t - H'\xi_{t|t-1})\\
K_t = P_{t|t-1}H(H'P_{t|t-1}H &+ R)^{-1}\\
\xi_{t|t} &= \xi_{t|t-1} + K_t\psi_t\\
P_{t|t} &= P_{t|t-1} - K_t H' P_{t|t-1}
\end{aligned}$$

Given the significant number of missing observations in the dataset, it is crucial to consider it in the recursion. Following Aruoba, Diebold, and Scotti (2009), when there are no observations in y_t, updating does not take place, and thus the algorithm becomes :

$$\xi_{t|t} = F\xi_{t-1|t-1} \tag{27}$$
$$P_{t|t} = FP_{t-1|t-1}F' + Q \tag{28}$$

Furthermore, let's assume that only a number n^* $(n > n^* > 0)$ of observations are available in y_t. Then, the measurement equation needs to be adapted as follows :

$$y_t^* = A'x_t^* + H^{*\prime}\xi_t + w_t^* \tag{29}$$

Where y_t^*, x_t^*, H^* and w_t^* are matrices and vectors with missing rows or columns corresponding to the missing observations. Thus, the transformation made in equation 24 corresponds to creating a matrix Γ_t which contains the n^* rows of I_n associated with those of y_t with an observed value such that :

$$\begin{aligned}
y_t^* &= \Gamma_t y_t \\
x_t^* &= \Gamma_t x_t \\
H^* &= \Gamma_t H \\
w_t^* &= \Gamma_t w_t \\
R_t^* &= \Gamma_t R_t \Gamma_t'
\end{aligned}$$

By performing those transformations, only the non-missing entries of y_t are kept and used in the updating stage of the algorithm. In the implementation, the three matrices in need of corrections due to missing observations are R_t $(n \times n)$, ψ_t $(n \times 1)$ and H_t $(r \times n)$. For instance, let's assume that at time t, the observation for the second observed measurement is missing $(n^* = n-1)$. Then, using the transformation described above, the corresponding rows and columns of R_t, ψ_t and H_t are removed. Hence, the new dimensions of the aforementioned matrices are : R_t $(n-1 \times n-1)$, ψ_t $(n-1 \times 1)$ and H_t $(r \times n-1)$. The procedure is analog when dealing with more than one missing observation.

The value of ξ_t is of interest given its structural interpretation: the Business Cycle Index. Therefore, the Kalman smoothing algorithm is used to obtain a better inference of the value of ξ_t based on the full set of data collected $\mathcal{Y}_T = (y_t, y_{t+1}, ..., y_T, x_t, x_{t+1}, ..., x_T)'$.
Such inference is called the "smoothed" estimate of ξ_t written as :

$$\xi_{t|T} \equiv E[\xi_t | \mathcal{Y}_T] \tag{30}$$

with MSE :

$$P_{t|T} \equiv E[(\xi_t - \xi_{t|T})(\xi_t - \xi_{t|T})'] \tag{31}$$

Once again, the methodology proposed by Hamilton (1994) is used to compute the smoothed estimates. As in any recursion, a starting value is needed, and for this purpose the Kalman filter is run first in order to acquire the sequences $\{\xi_{t|t}\}_{t=1}^{T}$, $\{\xi_{t+1|t}\}_{t=0}^{T-1}$, $\{P_{t|t}\}_{t=1}^{T}$ and $\{P_{t+1|t}\}_{t=0}^{T-1}$. Then, as a starting value for the algorithm, $\xi_{T|T}$ as the last entry in $\{\xi_{t|t}\}_{t=1}^{T}$ is chosen. The next stage consists of computing the sequence $\{J_t\}_{t=1}^{T-1}$ using :

$$J_t = P_{t|t} F' P_{t+1|t}^{-1} \tag{32}$$

Finally, the smoothed estimates are computed as follows :

$$\xi_{t|T} = \xi_{t|t} + J_t(\xi_{t+1|T} - \xi_{t+1|t}) \tag{33}$$

$$P_{t|T} = P_{t|t} + J_t(P_{t+1|T} - P_{t+1|t})J_t' \quad (34)$$

Proceeding backward through the sample in this fashion allows for the calculation of the complete set of smoothed estimates : $\{\xi_{t|T}\}_{t=1}^{T}$.

4 Maximum Likelihood Estimation

Thus far, it was assumed that the different matrices of parameters were known, which is evidently not appropriate for the purpose of this paper. Thankfully, the Kalman filter is perfectly equipped to address this issue. As a matter of fact, the computation of the log-likelihood is quite straightforward given the multivariate Gaussian density function :

$$f_Y(y_{1_i}, \cdots, y_{n_i}) = \frac{1}{(2\pi)^{\frac{n}{2}}|\Sigma|^{\frac{1}{2}}} e^{-\frac{1}{2}(y_i-\mu)'\Sigma^{-1}(y_i-\mu)}$$

$$\ln(\prod_{i=1}^{T} f_Y(y_{1_i}, \cdots, y_{n_i})) = -\frac{Tn}{2}\ln(2\pi) - \frac{T}{2}\ln(|\Sigma|) - \frac{1}{2}\sum_{i=1}^{T}(y_i - \mu)'\Sigma^{-1}(y_i - \mu)$$

Then, the log-likelihood can be computed as (taking into consideration that n may not be constant across periods because of missing observations) :

$$\begin{aligned}\ln(\prod_{i=1}^{T} f_{Y_t|X_t\mathcal{Y}_{t-1}}(y_t|x_t, \mathcal{Y}_{t-1})) = -\frac{1}{2}\sum_{i=1}^{T}(n_i * \ln(2\pi) + \ln(\det(H'P_{t|t-1}H + R)) \\ + (y_t - A'x_t - H'\hat{\xi}_{t|t-1})'(H'P_{t|t-1}H + R)^{-1}(y_t - A'x_t - H'\hat{\xi}_{t|t-1})\end{aligned}$$

Or using previous notations :

$$\begin{aligned}\ln(\prod_{i=1}^{T} f_{Y_t|X_t\mathcal{Y}_{t-1}}(y_t|x_t, \mathcal{Y}_{t-1})) = -\frac{1}{2}\sum_{i=1}^{T} n_i \ln(2\pi) + \ln(\det(V_{iy_{t|t-1}})) \\ + \psi_t'(V_{iy_{t|t-1}})^{-1}\psi_t \quad (35)\end{aligned}$$

Note that if all observations of y_t are missing then the contribution of period t to the log-likelihood will be equal to zero.

5 Data

The estimation problem faced to compute the Business Cycle Index is substantial. A significant number of observations is involved (especially for the daily Business Cycle Index), and a constraint regarding the number of coefficients that can be estimated with MLE requires to think the process through. We base the Business Cycle Index on a relatively small amount of underlying indicators all based on hard data (i.e. seven indicators) in order to reduce the number of parameters that need to be estimated by maximum likelihood. Note that the parameters are estimated individually for each of the 26 Swiss cantons.

5.1 Cantonal Data for the Monthly Business Cycle Index

The monthly cantonal variables are ordered first in the state-space representation and are the following: registered unemployed workers (1990M01-today, State Secretariat for Economic Affairs), industrial production (2010M01-today, own estimates), exports of goods (1995M01-today, Federal Office for Customs and Border Security), consumer spending (2011-today, Worldline and Monitoring Consumption) and vacancies (quarterly 1998M01-today, Swiss Job Tracker). The only quarterly stock variable is employment (1995Q3-today, own estimates) and is ordered sixth in the state-space representation. The only quarterly flow variable is real GDP (1997Q3-today, own estimates) which is also the target variable for the nowcasting exercise, and is ordered last in the state-space representation. All the variables are calendar- and outlier-adjusted using the X-13ARIMA-SEATS. As we work with year-on-year growth rates, we do not need to make a seasonal adjustment to the data.[2] Moreover, we check that very time-series is stationary by running a Dickey-Fuller test.[1]

5.2 Data for the Daily Business Cycle Index

The only daily variable is consumer spending and is ordered first in the state-space representation. The only weekly variable is the number of vacancies and is a stock variable. It is ordered second in the state-space representation. The only monthly stock variable is registered unemployed workers and is ordered third in the state-space representation. The monthly flow variables are industrial production and exports of goods. They are ordered fourth and fifth in the state-space representation. The quarterly stock variable is employment and is ordered sixth in the state-space representation. The only quarterly flow variable is real GDP and is ordered last in the state-space representation. When we move to the weekly index, consumer spending as well as vacancies are available at the weekly frequency and thus become weekly variables. All the variables are calendar- and outlier-adjusted using the X-13ARIMA-SEATS. As we work with year-on-year growth rates, we do not need to make a seasonal adjustment to the data.[3] Moreover, we check that very time-series is stationary by running a Dickey-Fuller test.[1]

[2]For operational reasons, we take time-series already adjusted by data providers.

[1]The null hypothesis being non-stationary, all the tests resulted in the rejection of the null hypothesis with a p-value smaller than 0.01.

[3]For operational reasons, we take time-series already adjusted by data providers.

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6 Model implementation

We take several further decisions to reduce the number of parameters that need to be estimated by maximum likelihood and to ease computation for signal extraction. Firstly, we make the simplifying assumption that the Business Cycle Index and the various observed variables follow first-order dynamics ($p = r = 1$). Secondly, we remove constants in measurement equations by standardizing all k-month (k-day) changes.[4] Thirdly, an Harvey Accumulator (Harvey, 1990) for each frequency $f \in \{w, m, q\}$ denoted by $Z_{f,t}$ is implemented to reduce the size of the state vector by summarizing the necessary information required to construct observed flow variables. It is defined as follows :

$$
\begin{aligned}
z_t^f &= \zeta_t^f z_{t-1}^f + b_t && (36)\\
&= \zeta_t^f z_{t-1}^f + \phi b_{t-1} + \epsilon_t && (37)
\end{aligned}
$$

where $\zeta_{f,t}$ is defined as :

$$
\zeta_t^f = \begin{cases} 0 & \text{if } t \text{ is the last day of the observational period } f \\ 1 & \text{otherwise.} \end{cases}
$$

6.1 Monthly Business Cycle Index

The equations that defines the monthly model are the state equation :

$$
\underbrace{\begin{bmatrix} b_{t+1} \\ z_{t+1}^m \end{bmatrix}}_{=\xi_{t+1}} = \underbrace{\begin{bmatrix} \phi & 0 \\ \phi & \zeta_t^m \end{bmatrix}}_{=F} \underbrace{\begin{bmatrix} b_t \\ z_t^m \end{bmatrix}}_{=\xi_t} + \underbrace{\begin{bmatrix} \epsilon_{t+1} \\ \epsilon_{t+1} \end{bmatrix}}_{=v_{t+1}}, \tag{38}
$$

and the observation equation :

$$
\underbrace{\begin{bmatrix} \Delta_k Y_{1,m_t} \\ \Delta_k Y_{2,m_t} \\ \Delta_k Y_{3,m_t} \\ \Delta_k Y_{4,m_t} \\ \Delta_{k(q)} Y_{5,q_t} \\ \Delta_{k(q)} Y_{6,q_t} \\ \Delta_{k(q)} Y_{7,q_t} \end{bmatrix}}_{=y_t} = \underbrace{\begin{bmatrix} \rho_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_7 \end{bmatrix}}_{=A'} \underbrace{\begin{bmatrix} \Delta_k Y_{1,m_t-1} \\ \Delta_k Y_{2,m_t-1} \\ \Delta_k Y_{3,m_t-1} \\ \Delta_k Y_{4,m_t-1} \\ \Delta_{k(q)} Y_{5,q_t-1} \\ \Delta_{k(q)} Y_{6,q_t-1} \\ \Delta_{k(q)} Y_{7,q_t-1} \end{bmatrix}}_{=x_t} + \underbrace{\begin{bmatrix} \lambda_1 & 0 \\ \lambda_2 & 0 \\ \lambda_3 & 0 \\ \lambda_4 & 0 \\ \lambda_5 & 0 \\ \lambda_6 & 0 \\ 0 & \lambda_6^* \end{bmatrix}}_{=H'} \underbrace{\begin{bmatrix} b_t \\ z_t^m \end{bmatrix}}_{=\xi_t} + \underbrace{\begin{bmatrix} u_{1,t} \\ u_{2,t} \\ u_{3,t} \\ u_{4,t} \\ u_{5,t} \\ u_{6,t} \\ u_{7,t}^* \end{bmatrix}}_{=w_t}, \tag{39}
$$

with :

$$
\begin{bmatrix} v_{t+1} \\ w_t \end{bmatrix} \sim N\left(\begin{bmatrix} 0_{2x1} \\ 0_{7x1} \end{bmatrix}, \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix} \right), \tag{40}
$$

[4] Since $\bar{b} = \bar{u}_i = \bar{u}_i^* = 0$ by assumption, it can be easily shown that all measurement equations remain valid after standardization.

where $Q = \text{diag}(1-\phi^2, 1-\phi^2)$ and $\text{R} = \text{diag}(\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2, \sigma_5^2, \sigma_6^2, \sigma_7^{2*})$.

6.2 Daily Business Cycle Index

The equations that defines the daily model are the state equation :

$$\underbrace{\begin{bmatrix} b_{t+1} \\ z_{t+1}^m \\ z_{t+1}^q \end{bmatrix}}_{=\xi_{t+1}} = \underbrace{\begin{bmatrix} \phi & 0 & 0 \\ \phi & \zeta_t^m & 0 \\ \phi & 0 & \zeta_t^q \end{bmatrix}}_{=F} \underbrace{\begin{bmatrix} b_t \\ z_t^m \\ z_t^q \end{bmatrix}}_{=\xi_t} + \underbrace{\begin{bmatrix} \epsilon_{t+1} \\ \epsilon_{t+1} \\ \epsilon_{t+1} \end{bmatrix}}_{=v_{t+1}}, \tag{41}$$

and the observation equation :

$$\underbrace{\begin{bmatrix} \Delta_{365} Y_{1,d_t} \\ \Delta_{52} Y_{2,w_t} \\ \Delta_{12} Y_{3,m_t} \\ \Delta_{12} Y_{4,m_t} \\ \Delta_{12} Y_{5,m_t} \\ \Delta_4 Y_{6,q_t} \\ \Delta_4 Y_{7,q_t} \end{bmatrix}}_{=y_t} = \underbrace{\begin{bmatrix} \rho_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_7 \end{bmatrix}}_{=A'} \underbrace{\begin{bmatrix} \Delta_{365} Y_{1,d_t-1} \\ \Delta_{52} Y_{2,w_t-1} \\ \Delta_{12} Y_{3,m_t-1} \\ \Delta_{12} Y_{4,m_t-1} \\ \Delta_{12} Y_{5,m_t-1} \\ \Delta_4 Y_{6,q_t-1} \\ \Delta_4 Y_{7,q_t-1} \end{bmatrix}}_{=x_t} + \underbrace{\begin{bmatrix} \lambda_1 & 0 & 0 \\ \lambda_2 & 0 & 0 \\ \lambda_3 & 0 & 0 \\ 0 & \lambda_4^* & 0 \\ 0 & \lambda_5^* & 0 \\ \lambda_6 & 0 & 0 \\ 0 & 0 & \lambda_7^* \end{bmatrix}}_{=H'} \underbrace{\begin{bmatrix} b_t \\ z_t^m \\ z_t^q \end{bmatrix}}_{=\xi_t} + \underbrace{\begin{bmatrix} u_{1,t} \\ u_{2,t} \\ u_{3,t} \\ u_{4,t}^* \\ u_{5,t}^* \\ u_{6,t} \\ u_{7,t}^* \end{bmatrix}}_{=w_t}, \tag{42}$$

with :

$$\begin{bmatrix} v_{t+1} \\ w_t \end{bmatrix} \sim N\left(\begin{bmatrix} 0_{3x1} \\ 0_{7x1} \end{bmatrix}, \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix} \right), \tag{43}$$

where $Q = \text{diag}(1-\phi^2, 1-\phi^2, 1-\phi^2)$ and $\text{R} = \text{diag}(\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^{2*}, \sigma_5^{2*}, \sigma_6^2, \sigma_7^{2*})$.

6.3 Daily Business Cycle Index with Weekly Data Only

The equations that defines the daily model are the state equation :

$$\underbrace{\begin{bmatrix} b_{t+1} \\ z_{t+1}^w \\ z_{t+1}^m \\ z_{t+1}^q \end{bmatrix}}_{=\xi_{t+1}} = \underbrace{\begin{bmatrix} \phi & 0 & 0 & 0 \\ \phi & \zeta_t^w & 0 & 0 \\ \phi & 0 & \zeta_t^m & 0 \\ \phi & 0 & 0 & \zeta_t^q \end{bmatrix}}_{=F} \underbrace{\begin{bmatrix} b_t \\ z_t^w \\ z_t^m \\ z_t^q \end{bmatrix}}_{=\xi_t} + \underbrace{\begin{bmatrix} \epsilon_{t+1} \\ \epsilon_{t+1} \\ \epsilon_{t+1} \\ \epsilon_{t+1} \end{bmatrix}}_{=v_{t+1}}, \tag{44}$$

and the observation equation :

$$\underbrace{\begin{bmatrix} \Delta_{52}Y_{1,w_t} \\ \Delta_{52}Y_{2,w_t} \\ \Delta_{12}Y_{3,m_t} \\ \Delta_{12}Y_{4,m_t} \\ \Delta_{12}Y_{5,m_t} \\ \Delta_{4}Y_{6,q_t} \\ \Delta_{4}Y_{7,q_t} \end{bmatrix}}_{=y_t} = \underbrace{\begin{bmatrix} \rho_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_7 \end{bmatrix}}_{=A'} \underbrace{\begin{bmatrix} \Delta_{52}Y_{1,d_t-1} \\ \Delta_{52}Y_{2,w_t-1} \\ \Delta_{12}Y_{3,m_t-1} \\ \Delta_{12}Y_{4,m_t-1} \\ \Delta_{12}Y_{5,m_t-1} \\ \Delta_{4}Y_{6,q_t-1} \\ \Delta_{4}Y_{7,q_t-1} \end{bmatrix}}_{=x_t} + \underbrace{\begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2^* & 0 & 0 \\ \lambda_3 & 0 & 0 & 0 \\ 0 & 0 & \lambda_4^* & 0 \\ 0 & 0 & \lambda_5^* & 0 \\ \lambda_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_7^* \end{bmatrix}}_{=H'} \underbrace{\begin{bmatrix} b_t \\ z_t^w \\ z_t^m \\ z_t^q \end{bmatrix}}_{=\xi_t} + \underbrace{\begin{bmatrix} u_{1,t} \\ u_{2,t} \\ u_{3,t} \\ u_{4,t}^* \\ u_{5,t}^* \\ u_{6,t} \\ u_{7,t}^* \end{bmatrix}}_{=w_t}, \quad (45)$$

with :

$$\begin{bmatrix} v_{t+1} \\ w_t \end{bmatrix} \sim N\left(\begin{bmatrix} 0_{3x1} \\ 0_{7x1} \end{bmatrix}, \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix}\right), \quad (46)$$

where $Q = \text{diag}(1-\phi^2, 1-\phi^2, 1-\phi^2)$ and R $= \text{diag}(\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^{2*}, \sigma_5^{2*}, \sigma_6^2, \sigma_7^{2*})$.

6.4 Model estimation

Using homemade R codes and this methodology, the estimation process is described below. First, start-up coefficients' values for the stock variables and flow variables at the frequency of the Business Cycle Index are estimated by running the Kalman filter once without using flow variables at a lower frequency than the index. The first step, which was just presented, provides the first estimate for all the coefficients of stock variables and flow variables at the frequency of the Business Cycle Index in the observation equation and all the coefficients of the state equation. Moreover, a first estimation of the business cycle b_t^* is extracted.

Then, to obtain initial values for the coefficients associated with the remaining flow variables in the observation equation, a simple OLS regression of the flow variables on the first estimate of the business cycle and the lag of the said variables is run.

$$y_t^i = \lambda^i b_t^* + \rho^i y_{t-1}^i + \epsilon_t^i \quad (47)$$

At the end of this second stage, initial values for all the parameters of the model are available. Finally, an estimation using all the model's coefficients jointly is made before extracting the final smoothed estimate of the Business Cycle Index.

It is worth noting that to perform the maximum likelihood estimation, two optimization algorithms are used alternatively to increase the chances of reaching a global maximum rather than a local one. This is due to algorithms tending to identify a path in which they will be locked until convergence. Thus, using two algorithms that do not use the same method alleviate this problem. The two algorithms used are part of the optimx R package and are named "nlminb" and "Nelder-Mead".

Appendix

A.5 Data

Most of the data used to extract the BCI are expressed in real terms, except cantonal exports and consumer spending at the national and cantonal levels. We deflate these variables in the following way.

Comuputing deflator for cantonal exports. We compute the national deflator for exports of goods as $P_t^x = \frac{X_t^{nom}}{X_t^{real}}$, where X_t are data provided by the The Federal Office for Customs and Border Security. We assume that $P_t^x = P_{c,t}^x$ to to express in real terms cantonal exports. This is a good approximation since (1) the underlying data are the same at the cantonal and national level; and (2) the underlying macro shocks driving the dynamics of the deflator should be the same at the cantonal and national level. Indeed, we are not interested in the evolution of real cantonal exports and not their levels.

Comuputing deflator for consumer spending. Data on consumer spending from Worldline and Monitoring Consumption Switzerland are available for 12 categories. We match each of these categories with monthly price data available from the Swiss Federal Statisitical Office (SFSO):

Table xxx: Correspondance MCS

NOGA	Merchant category	Statistics SFSO	ID	Category	Weight 201
4719 / 474-479	Retail: Other goods	Retail Trade Turnover Statistics	NOGA 4719 / 474-479	Retail sale of non-food	36.1
4711 / 472	Retail: Food, beverage, tobacco	Swiss Consumer Price Index	COICOP 100-1	Food and non-alcoholic beverages	29.2
56	Food and beverage services	Swiss Consumer Price Index	COICOP 100-11001	Catering services	7.8
473	Retail: Fuel stations	Swiss Consumer Price Index	COICOP 100-7105	Fuel	7.8
55	Accommodation	Swiss Consumer Price Index	COICOP 100-11170	Accommodation	5.6
-	Professional services	Swiss Consumer Price Index	COICOP 110-102	Services	3.3
49-51	Transport services	Swiss Consumer Price Index	COICOP 100-7200	Transport services	3.3
90-93	Entertainment and sports	Swiss Consumer Price Index	COICOP 100-9350	Recreational and cultural services	2.2
94-96	Personal services	Swiss Consumer Price Index	COICOP 100-12	Other goods and services	1.5
86	Human health services	Swiss Consumer Price Index	COICOP 100-6	Healthcare	1.5
45	Motor vehicles	Swiss Consumer Price Index	COICOP 100-7002	Purchase of cars motorcycles, bicycles	1.1
-	Other	Swiss Consumer Price Index	COICOP 100-100	Total	0.8

Notes: bla bla bla.

For the category of non-food retail, we use the deflator computed from the Retail Trade Turnover Statistics even though the publication delay is greater than with Swiss Consumer price index (i.e approximately 4 months). For the category of food, beverage and tobacco from the retail sector, we use the consumer price index that that does not include alcoholic beverages and tobacco since this aggregate does not exist.[5] For the category of Motor vehicles, we use the consumer price index without use and maintenance of cars, motorcycles, bicycles since it include the volatile component of fuel. Based the input-output tables, the category of Personal services is matched with COICOP 12 "'Other goods and services"'.

[5]The weight in the Swiss CPI is equal is much larger for Food and non-alcoholic beverages (11.0%) than for Alcoholic beverages and tobacco (2.9%).

The deflator used to express data of consumer spending from Worldline and Monitoring Consumption Switzerland in real terms is computed as follows:

$$P_t = \sum_{i=1}^{12} \omega_{i,t} P_{i,m_t}$$

where $\omega_{i,t}$ is the time-t weight associated with merchant category i and P_{i,m_t} is the monthly price index taken from Table xxx. When data are not avalaible, we use an ARIMA model to forecast prices. Note that the weights are computed at the observational frequency of data on concumer spending since there is a strong seasonal pattern. The deflator is seasonally adjusted in a second step.

The same deflator is used for the national BCI and all cantonal BCI (i.e. $P_{c,t} = P_t$) because data on weights are not available for the cantons (neither for prices).

References

[1] K. Abberger, M. Graff, B. Siliverstovs, and J.-E. Sturm. The kof economic barometer, version 2014. KOF Working papers 14-353, KOF Swiss Economic Institute, ETH Zurich, 2014.

[2] F. Altissimo, A. Bassanetti, R. Cristadoro, M. Forni, M. Lippi, L. Reichlin, and G. Veronese. Eurocoin: A real time coincident indicator of the euro area business cycle. *European Economics eJournal*, 2001.

[3] S. B. Aruoba, F. X. Diebold, and C. Scotti. Real-time measurement of business conditions. *Journal of Business Economic Statistics*, 27(4):417–427, 2009.

[4] C. Baumeister, D. Leiva-León, and E. Sims. Tracking weekly state-level economic conditions. *Review of Economics and Statistics*, pages 1–45, 2022.

[5] J. Boivin and S. Ng. Are more data always better for factor analysis? *Journal of Econometrics*, 132(1):169–194, 2006.

[6] A. F. Burns and W. C. Mitchell. Measuring business cycles. *National Bureau of Economic Research*, 1946.

[7] M. Burri and D. Kaufmann. A daily fever curve for the swiss economy. *Swiss Journal of Economics and Statistics*, 156(1):1–11, 2020.

[8] M. Camacho and G. Perez-Quiros. Introducing the euro-sting: Short-term indicator of euro area growth. *Journal of Applied Econometrics*, 25(4):663–694, 2010.

[9] M. Camacho and G. Perez-Quiros. Introducing the euro-sting: Short-term indicator of euro area growth. *Journal of Applied Econometrics*, 25(4):663–694, 2010.

[10] M. Chauvet. An econometric characterization of business cycle dynamics with factor structure and regime switching. *International Economic Review*, 39(4):969–996, 1998.

[11] R. B. Cleveland, W. S. Cleveland, J. E. McRae, and I. Terpenning. Stl: A seasonal-trend decomposition procedure based on loess (with discussion). *Journal of Official Statistics*, 6:3–73, 1990.

[12] F. X. Diebold. Real-time real economic activity: Exiting the great recession and entering the pandemic recession. Working Paper 27482, National Bureau of Economic Research, July 2020.

[13] F. X. Diebold and R. S. Mariano. Comparing predictive accuracy. *Journal of Business & Economic Statistics*, 20(1):134–144, 2002.

[14] F. X. Diebold and G. D. Rudebusch. Measuring business cycles: A modern perspective. *The Review of Economics and Statistics*, 78(1):67–77, 1996.

[15] J. Durbin and S. J. Koopman. *Time Series Analysis by State Space Methods*. Oxford University Press, 2 edition, 2012.

[16] M. D. D. Evans. Where Are We Now? Real-Time Estimates of the Macroeconomy. *International Journal of Central Banking*, 1(2), September 2005.

[17] M. Forni, M. Hallin, M. Lippi, and L. Reichlin. The generalized dynamic-factor model: Identification and estimation. *The Review of Economics and Statistics*, 82(4):540–554, 2000.

[18] A. Galli. Which Indicators Matter? Analyzing the Swiss Business Cycle Using a Large-Scale Mixed-Frequency Dynamic Factor Model. *Journal of Business Cycle Research*, 14(2):179–218, November 2018.

[19] J. D. Hamilton. *Time Series Analysis*. Princeton University Press, 1 edition, Jan. 1994.

[20] A. C. Harvey. *Forecasting, structural time series models, and the Kalman filter / Andrew Harvey*. Cambridge University Press Cambridge ; New York, 1990.

[21] R. Hodrick and E. Prescott. Postwar u.s business cycles: An empirical investigation. *Journal of Money, Credit and Banking*, 29(1):1–16, Feb. 1997.

[22] R. Mariano and Y. Murasawa. A coincident index, common factors, and monthly real gdp. *Oxford Bulletin of Economics and Statistics*, 72:27–46, 02 2010.

[23] R. S. Mariano and Y. Murasawa. A new coincident index of business cycles based on monthly and quarterly series. *Journal of Applied Econometrics*, 18(4):427–443, 2003.

[24] T. Proietti and F. Moauro. Dynamic factor analysis with non-linear temporal aggregation constraints. *Journal of the Royal Statistical Society. Series C (Applied Statistics)*, 55(2):281–300, 2006.

[25] M. Ravn and H. Uhlig. On adjusting the hodrick-prescott filter for the frequency of observations. *The Review of Economics and Statistics*, 84(2):371–375, 2002.

[26] J. Stock and M. Watson. New indexes of coincident and leading economic indicators. In *NBER Macroeconomics Annual 1989, Volume 4*, pages 351–409. National Bureau of Economic Research, Inc, 1989.

[27] J. Stock and M. W. Watson. *A Probability Model of the Coincident Economic Indicators*, pages 63–90. Cambridge University Press, 1991.

[28] J. H. Stock and M. W. Watson. Macroeconomic forecasting using diffusion indexes. *Journal of Business & Economic Statistics*, 20(2):147–162, 2002.